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Modification of The Leslie Model on Population Growth in The Bangka Belitung Islands Province

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Abstract
The Leslie model is one of the applications of Algebra in solving a population growth model. The birth rate and survival rate of a population are constituents of the Leslie Matrix. The advantage of this model is that it only requires data on the total female population. This study aims to modify the classic Leslie Model by adding correction values to matrix elements, especially birth rates and survival rates. The correction value is obtained from the minimum Euclidean distance for each birth rate and survival rate for each population age group. The Euclidean distance is used because it requires simple calculations and often used for grouping data (clustering). The more similar data so, the closer the Euclidean distance between the two data. Based on the modified results, the Perron value obtained from the Leslie matrix is 0,9. There are differences in prediction results between the classic Leslie model in the Bangka Belitung population growth model and the modified Leslie model. Leslie's modification is closer to the actual population of the Bangka Belitung Islands Province.

Keywords: *euclid distance, leslie model, demography, population*

1. Introduction

The ever-increasing population is one of the advantages as well as a problem in Indonesia. The results of the 2020 population census recorded a population of 270,20 million in Indonesia. This figure increased by 36,56 million compared to the 2010 population census. A large population is certainly the strength of a country, but it can also cause problems if it is not handled properly. Government policies in terms of subsidies, per capita income, poverty levels and many other things are strongly influenced by population growth [1].

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The Bangka Belitung archipelago province is a relatively young province in Indonesia. Based on BPS data for 2021, the population of Bangka Belitung is around 1.517.590. The Department of Population and Civil Registry shows that 51,26% of Babylon's population are male and 48,74% are female. If seen from the data available at BPS, the population of Bangka Belitung continues to increase by around 1,9% every year. This figure is higher than Indonesia's growth rate in general, which is around 1,25%. Ideally to reduce Indonesia's population density, the percentage increase in population is expected to be around 0,5% [2].

Demography is the study of population composition. In demography, mathematics is needed in analyzing formal demography [3]. Modeling is a part of Mathematics to describe a situation that exists in the real world to a Mathematical model or a formula.[3], [4]. One of the methods for determining the population is the Leslie Model discovered by P.H. Leslie in 1945. This model requires data on the total female population to obtain birth rates and survival rates which are then used to form the Leslie Matrix. Several studies have used this model to calculate the population [5]-[9].In mathematical modeling, data discrepancies are often found or even data is not available for the problem to be solved. Therefore, other methods will be used to correct the lack of data. In this study, Euclidean distance will be used. The Euclidean distance is often used in facial imaging [10], determine the proportion of employees [11].

In previous research, modeling of the population of the Bangka Belitung Islands Province was carried out [4]. However, there are limited data in determining the birth rate according to a certain age group or what is known as the Age Specific Fertility Rate (ASFR). Furthermore, in this study, we will look for correction values for birth rates using the Euclidean distance for each age group, and we will also look for correction rates for survival. The corrected values obtained will then be used to construct a new Leslie Model.

Based on the explanation above, in this research a modification of the Leslie model will be carried out from the model that has been found in previous research in the Bangka Belitung Islands Province [4]. Modifications are applied to birth rates and survival rates by adding correction numbers obtained from the minimum Euclidean distance for each age group.

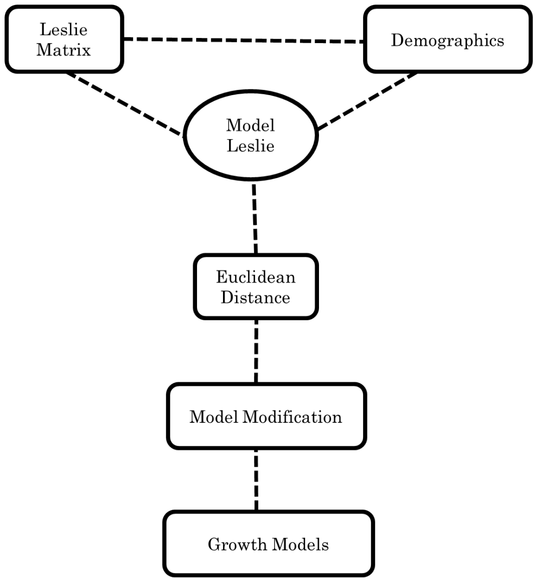
2. Method

The method used in this research is literature and applied studies. The first thing the researcher did was to review some of the literature on the Matrix, the Leslie Matrix, the use of the Leslie Matrix in demography, and some literature on the development of the Leslie Matrix. Furthermore, it is studied about the concept of Euclidean distance which will be used in the Leslie Matrix so that a modification of the Leslie model is obtained. Furthermore, the modification of the Leslie model will be applied in predicting the population of the Bangka Belitung Islands using secondary data and the help of the Matematica 5.0 software. Briefly the research design is presented in Figure 1.

2.1. **Leslie Matrix.** The Leslie matrix is a square matrix formed by the survival rate and birth rate of a population. The advantage of this Leslie Matrix is that the data needed is only data from the female population. Furthermore, from the resulting matrix, the Leslie model will be obtained.

If $p_1, p_2, \cdots, p_n$ is the birth rate for each age group and $q_1, q_2, \cdots, q_{n-1}$ is the survival rate for each age group, the Leslie matrix can be formed as follows

$$L = \begin{bmatrix} p_1 & p_2 & p_3 & \cdots & p_{n-1} & p_n \\ q_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & q_n & 0 \end{bmatrix} \tag{1}$$



Gambar 1. Research Design

Suppose λ is the root of the perron or the largest positive eigenvalue of the matrix L, then λ can replace the position of L in the Leslie model. This property is guaranteed by the following theorem,

Teorema 2.1. *If λ is the perron root of the Leslie matrix L then $X^k = LX^{k-1}$ can be represented by $X^k = \lambda X^{k-1}$ where X^k is the number of women in the t_k age group and X^{k-1} is the number of women in the t_{k-1} age group.[12]*

Proof. The eigenvalues of the Leslie Matrix can be found using the characteristic equation $\lambda = det\,(L - \lambda I)$. Leslie matrix L is diagonalized, then $L = VDV^{-1}$ where D is a diagonal matrix whose elements are the eigenvalues of L and V is the correspondence of the eigenvectors. So that,

$$L = V \begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \lambda_n \end{bmatrix} V^{-1}$$

$$L^k = \left(VDV^{-1}\right)^k = \underbrace{\left(VDV^{-1}\right)\left(VDV^{-1}\right)\cdots\left(VDV^{-1}\right)}_{kfactor} = VD^kV^{-1}$$

$$L^k = V \begin{bmatrix} \lambda_1^k & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^k & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \lambda_n^k \end{bmatrix} V^{-1}$$

For any age vector x^0 we can find the age vector x^k after k years by finding L^kx^0

$$x^k = L^kx^0 = V \begin{bmatrix} \lambda_1^k & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^k & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \lambda_n^k \end{bmatrix} V^{-1}x^0$$

$$x^k = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} \lambda_1^k & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2^k & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \lambda_n^k \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ 0 \\ c_n \end{bmatrix}, V^{-1}x^0 = \begin{bmatrix} c_1 \\ c_2 \\ 0 \\ c_n \end{bmatrix}$$

$$x^k = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \begin{bmatrix} c_1\lambda_1^k \\ c_2\lambda_2^k \\ \vdots \\ c_n\lambda_n^k \end{bmatrix}$$

So that

$$x^k = c_1\lambda_1^k v_1 + c_2\lambda_2^k v_2 + \cdots + c_n\lambda_n^k v_n$$

Suppose λ_1^k is the largest eigenvalue of any existing eigenvalues

$$\frac{x^k}{\lambda_1^k} = \frac{\lambda_1^k}{\lambda_1^k}c_1v_1 + \frac{\lambda_2^k}{\lambda_1^k}c_2v_2 + \cdots + \frac{\lambda_n^k}{\lambda_1^k}c_nv_n$$

$$\lim_{k\to\infty}\left(\frac{x^k}{\lambda_1^k}\right) = \lim_{k\to\infty}\left(\frac{\lambda_1^k}{\lambda_1^k}c_1v_1 + \frac{\lambda_2^k}{\lambda_1^k}c_2v_2 + \cdots + \frac{\lambda_n^k}{\lambda_1^k}c_nv_n\right)$$

$$= c_1v_1 + \lim_{k\to\infty}\left(\frac{\lambda_2^k}{\lambda_1^k}c_2v_2 + \frac{\lambda_3^k}{\lambda_1^k}c_3v_3 + \cdots + \frac{\lambda_n^k}{\lambda_1^k}c_nv_n\right)$$

$$= c_1v_1$$

It means $\left|\frac{x^k}{\lambda_1^k} - c_1v_1\right| < \varepsilon$ and $0 < \frac{x^k}{\lambda_1^k} - c_1v_1 < \varepsilon$
So that $\frac{x^k}{\lambda_1^k} - c_1v_1 = 0$ and for large values of k the approximate value for x^k up to $x^k \approx \lambda_1^k c_1 v_1$.
For $k-1$ applies $x^{k-1} \approx \lambda_1^{k-1} c_1 v_1$.
We can write

$$x^k \approx \lambda_1^k c_1 \frac{x^{k-1}}{\lambda_1^{k-1} c_1}$$

$$x^k \approx \lambda_1 x^{k-1}$$

□

2.2. **Euclidean Distance.** Euclidean distance is the most common type of measurement used for grouping data. The more similar the data, the closer the Euclidean distance between the two data. The following is a definition of the Euclidean distance.
If x and y are vectors on R^n, the Euclidean distance can be defined as follows [11]

Definisi 2.2. *The Euclidean distance between the* $x = (x_1, x_2, \cdots, x_n)$ *and* $y = (y_1, y_2, \cdots, y_n)$ *are*

$$d(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + \cdots + (x_n - y_n)^2} \tag{2}$$

In this research, the Euclidean distance is used to find the shortest distance from the data used.

3. Results and Discussion

The Leslie model is modified using the Euclidean distance for birth rates for each age group. For example, $p_1, p_2, p_3, \cdots, p_n$ represents the birth rate of babies in each age group, but previously assumed $p_1 = (x_1, y_1), p_2 = (x_2, y_2), p_3 = (x_3, y_3), \cdots, p_n = (x_n, y_n)$
Next, find the Euclidean distance from the birth rate, namely

$$d(p_i, p_j) = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}, p_i \neq 0 \text{and } p_j \neq 0$$

Choose $\varepsilon = mind(p_i, p_j)$
Then the survival rate for each age group is denoted as $q_1, q_2, q_3, \cdots, q_n$ and assume $q_1 = (v_1, w_1), q_2 = (v_2, w_2), q_3 = (v_3, w_3), \cdots, q_n = (v_n, w_n)$. The Euclidean distance for survival is

$$d(q_i, q_j) = \sqrt{(v_i - w_j)^2 + (v_i - w_j)^2}, q_i \neq 0 \text{and } q_j \neq 0$$

Next choose $\delta = mind(q_i, q_j)$.

The value of ε will then be the corrected value for the birth rate for each age group, provided that $p_i \neq 0, i = 1, 2, 3, \cdots, n$. While the value of δ will be the corrected value with the condition $q_i < 1, i = 1, 2, 3, \cdots, n$ because the maximum value is 1.

$$L = \begin{bmatrix} p_1 & p_2 & p_3 & \cdots & p_{n-1} & p_n \\ q_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & q_n & 0 \end{bmatrix}$$

Modified to

$$L* = \begin{bmatrix} p_1 + \varepsilon & p_2 + \varepsilon & p_3 + \varepsilon & \cdots & p_{n-1} + \varepsilon & p_n + \varepsilon \\ q_1 + \delta & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2 + \delta & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & q_n + \delta & 0 \end{bmatrix}$$

This modification of the Leslie model will be applied to population growth data for the Bangka Belitung Islands Province, along with a table of birth rates and survival rates for 16 age groups. But before that, the following data on the birth rate of survival before being modified.

TABEL 1. Birth Rate and Survival Rate of the Bangka Belitung Islands Province

Age Group	Birth Rate	Survival Rate
0 to 4	0	0,97
5 to 9	0	0,97
10 to 14	0	0,99
15 to 19	0,041	1,0
20 to 24	0,117	1,0
25 to 29	0,13	0,98
30 to 34	0,105	0,93
35 to 39	0,061	0,91
40 to 44	0,022	0,86
45 to 49	0,006	0,85
50 to 54	0	0,83
55 to 59	0	0,76
60 to 64	0	0,75
65 to 69	0	0,67
70 to 74	0	1,0
75+	0	0,00

Source:BPS and BKKBN

Based on the table above, the Leslie Matrix is obtained as follows

$L =$

$$\begin{bmatrix}
0 & 0 & 0 & 0,041 & 0,117 & 0,13 & 0,105 & 0,061 & 0,022 & 0,006 & 0 & 0 & 0 & 0 & 0 & 0 \\
0,97 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0,97 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0,99 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0,98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0,93 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,91 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,86 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,85 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,83 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,76 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,75 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0,67 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}$$

The perron value of L is $\lambda = 0,88$
The population growth model is obtained as follows

$$X^k = 0,88X^{k-1}, k = 1,2,3,\cdots,n\,(3) \tag{3}$$

Based on model (3) the prediction of the population of the Bangka Belitung Islands Province is obtained as

Following

Tabel 2. Population Prediction Using the Leslie Model

Year	Number of Women	Population Prediction
2013	632.470	1.159.528
2014	645.984	1.184.304
2015	659.590	1.209.248
2016	673.247	1.234.286
2017	686.934	1.259.379
2018	700.618	1.284.466
2019	714.269	1.309.493
2020	706.130	1.294.572
2021	715.613	1.311.957
2022	728.179	1.334.995

Furthermore, model (3) above will be modified by adding the correction values obtained from the Euclidean distance, namely $\varepsilon = 0,012$ and $\delta = 0,01$, so that it is obtained

Tabel 3. Birth Rate and Survival Rate of the Bangka Belitung Islands Province

Age Group	Birth Rate	Survival Rate
0 to 4	0	0,99
5 to 9	0	0,98
10 to 14	0	0,1
15 to 19	0,053	1,0
20 to 24	0,129	1,0
25 to 29	0,142	0,99
30 to 34	0,117	0,94
35 to 39	0,073	0,92
40 to 44	0,034	0,87
45 to 49	0,018	0,86
50 to 54	0	0,84
55 to 59	0	0,77
60 to 64	0	0,76
65 to 69	0	0,68
70 to 74	0	1,0
75+	0	0,00

Based on the table above, the Leslie Matrix is formed as follows

$$L =$$

$$\begin{bmatrix}
0 & 0 & 0 & 0{,}041+\varepsilon & 0{,}117+\varepsilon & 0{,}13+\varepsilon & 0{,}105+\varepsilon & 0{,}061+\varepsilon & 0{,}022+\varepsilon & 0{,}006+\varepsilon & 0 & 0 & 0 & 0 & 0 & 0 \\
0{,}97+\delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0{,}97+\delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0{,}99+\delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0{,}98+\delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0{,}93+\delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}91+\delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}86+\delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}85+\delta & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}83+\delta & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}76+\delta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}75+\delta & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}67+\delta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}$$

$$L* = \begin{bmatrix} 0 & 0 & 0 & 0{,}053 & 0{,}129 & 0{,}142 & 0{,}117 & 0{,}073 & 0{,}034 & 0{,}018 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0{,}98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0{,}98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0{,}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0{,}99 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0{,}94 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}92 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}87 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}86 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}84 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}77 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}76 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}68 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The eigenvalues obtained from $L*?$ are a $\lambda* = 0,9$

So that the modification of the Leslie Model becomes,

$$X^k = 0,9X^{k-1}, k = 1, 2, 3, \cdots, n \, (4) \tag{4}$$

Based on the Leslie model above, the prediction of the population of the Bangka Belitung Islands Province is as follows

Tabel 4. Population Prediction Using Modified Leslie Model

Year	Number of Women	Population Prediction
2013	632.470	1.185.881
2014	645.984	1.211.220
2015	659.590	1.236.731
2016	673.247	1.262.338
2017	686.934	1.288.001
2018	700.618	1.313.659
2019	714.269	1.339.254
2020	706.130	1.323.994
2021	715.613	1.341.774
2022	728.179	1.365.336

The results of the modification of the Leslie model above, the following results are obtained

Lemma 3.1. *If $\lambda*$ is the perron root of the Leslie matrix $L*?$, then $X^k = L * X^{k-1}$ can be represented by $X^k = \lambda * X^{k-1}$ where X^k is the number of women in the age group t_k and X^{k-1} is the number of women in the age group t_{k-1}.*

Proof. By using Theorem 1 then it's clear that

$$X^k = LX^{k-1}$$

can be represented by

$$X^k = \lambda X^{k-1}$$

So that,

$$X^k = L * X^{k-1}$$

can be represented by

$$X^k = \lambda * X^{k-1}$$

□

Furthermore, the modified Lelie model is a generalization of the Leslie model because if the values $\varepsilon = mind\,(p_i, p_j) = 0$ and $\delta = mind\,(q_i, q_j) = 0$, the modified Leslie model will return to the initial Leslie model. This property is stated in the following Lemma,

Lemma 3.2. *If* $\varepsilon = mind\,(p_i, p_j) = 0$ *and* $\delta = mind\,(q_i, q_j) = 0$ *then* $L* = L$.

Proof. Let $L* = \begin{bmatrix} p_1+\varepsilon & p_2+\varepsilon & p_3+\varepsilon & \cdots & p_{n-1}+\varepsilon & p_n+\varepsilon \\ q_1+\delta & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2+\delta & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & q_n+\delta & 0 \end{bmatrix}$

We have $\varepsilon = mind\,(p_i, p_j) = 0$ and $\delta = mind\,(q_i, q_j) = 0$

So that,

$$
\begin{aligned}
L* &= \begin{bmatrix} p_1+\varepsilon & p_2+\varepsilon & p_3+\varepsilon & \cdots & p_{n-1}+\varepsilon & p_n+\varepsilon \\ q_1+\delta & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2+\delta & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & q_n+\delta & 0 \end{bmatrix} \\
&= \begin{bmatrix} p_1+0 & p_2+0 & p_3+0 & \cdots & p_{n-1}+0 & p_n+0 \\ q_1+0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2+0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & q_n+0 & 0 \end{bmatrix} \\
&= \begin{bmatrix} p_1 & p_2 & p_3 & \cdots & p_{n-1} & p_n \\ q_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & q_2 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & q_n & 0 \end{bmatrix} \\
&= L
\end{aligned}
$$

□

4. Conclusion

Based on the results and discussion above, it can be concluded that from the Euclidean distance used, a corrected birth rate of 0,012 and a survival rate of 0,01 is obtained, resulting in a modified Leslie matrix which has a perron value of 0,9. There are differences in prediction results between the classic Leslie model in the Bangka Belitung population growth model and the modified Leslie model. Leslie's modification is closer to the actual population of the Bangka Belitung Islands Province.

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