Jurnal Matematika Integratif.
Vol. 22, No. 1 (2026), pp. 101–118.

p-ISSN:1412-6184, e-ISSN:2549-903
doi:10.24198/jmi.v22.n1.69207.101-118

Application of Spatial Weight Matrix based on Semivariogram in Space-Time Autoregressive Integrated (STARI) Model for Financial System Forecasting in the Greater Bandung Region

Yurid Audina[1*], Budi Nurani Ruchjana[2], and Atje Setiawan Abdullah[3]

[1,2]Department of Mathematics, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran
[3]Department of Computer Science, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran
Jl. Raya Bandung Sumedang KM.21, Hegarmanah, Kec. Jatinangor,
Kabupaten Sumedang, Jawa Barat 45363, Indonesia

Email: *yurid23001@mail.unpad.ac.id

Abstract

The global financial crisis of 2022 had a significant impact on the stability of Indonesia's financial sector, marked by fiscal expansion and an increase in the money supply. The uneven distribution of liquidity across regions generated disparities in regional inflation, resulting in macroeconomic dynamics that exhibited a complex spatio-temporal structure. These conditions require a forecasting approach capable of capturing spatial and temporal interactions simultaneously. This study applies the Space-Time Autoregressive Integrated (STARI) model to describe monthly inflation dynamics that are non-stationary due to inter-regional trends within Bandung Raya area. Spatial dependence is represented through spatial weight matrices constructed using three approaches matrices: uniform weights, inverse-distance weights, and isotropic semivariogram weights derived from population density data. Their effects on forecasting accuracy are compared using the Mean Squared Error (MSE). The novelty of the proposed approach lies in the use of an isotropic semivariogram as the basis for constructing spatial weights, allowing the model to capture continuous and heterogeneous spatial autocorrelation beyond traditional distance-based methods. Model parameters are estimated using Ordinary Least Squares (OLS) method implemented through Python scripts, and model evaluation is conducted using forecasting accuracy criteria and error diagnostics. The results indicate that the STARI(1,1,1) model incorporating semivariogram-based spatial weights outperforms both uniform and inverse-distance weights in terms of forecasting accuracy, because it has a minimum MSE. These findings provide valuable insights for economic policy formulation in Bandung Raya area.

Keywords: *STARI, inverse distance weight matrix, semivariogram, inflation, MSE.*

2020 Mathematics Subject Classification: 62M10, 62M30
Received: 12-01-2026, accepted: 28-05-2026.

1. Introduction

The 2022 global financial crisis led to increased government debt in many countries, including Indonesia, and put pressure on economic growth and financial stability. To respond to the risk of recession, the Indonesian government implemented a fiscal expansion policy by increasing debt financing to stimulate national economic recovery [1]. This policy was also aimed at supporting the achievement of the Sustainable Development Goals (SDGs), particularly goals 1, 8, 9, and 10. However, the effectiveness of fiscal expansion varies across regions due to spatial characteristics such as population density and local economic structure [5], as well as temporal dynamics related to the timing of policy implementation and regional economic cycles [4].

The impact of this fiscal expansion is reflected in an increase in the money supply, which has the potential to cause inflationary pressures [3]. Bank Indonesia data for 2024 shows that all components of the money supply, both M1 (inflation) and M2 (money supply), experienced significant increases. Money in circulation increased from IDR 8,739.6 trillion in February to IDR 8,888.4 trillion in March 2024, with annual growth reaching 7.2 percent. The highest increase occurred in non-bank currency, while several savings components actually contracted. On the other hand, West Java inflation data shows considerable variation between regions, with Bandung Regency recording higher inflation than Bandung City in different periods [2]. This condition indicates that inflation dynamics are spatio-temporal, influenced by the simultaneous interaction of time and location factors [10].

Previous research supports the existence of spatial dependence in inflation dynamics. Inflation has been shown to have spatial spillover effects between regions using the Spatial Autoregressive (SAR) approach, resulting in an uneven geographical distribution of price fluctuations [9]. Meanwhile, the Geographically Weighted Regression (GWR) approach was used to find that the influence of monetary variables on inflation differed across districts/cities in Java [15]. Inflation data was evaluated using the Autoregressive Moving Average (ARIMA) and Vector Autoregressive (VAR) models [17]. These results emphasize the importance of approaches capable of simultaneously capturing spatial and temporal interactions in regional inflation analysis. One relevant spatio-temporal approach is the Space-Time Autoregressive (STAR) model, which combines spatio-temporal models through time lags and spatial lags using a spatial weight matrix [12]. This model was later developed into the Space-Time Autoregressive Integrated (STARI) model to accommodate non-stationary data with trends. Further development was carried out through the Generalized Space-Time Autoregressive (GSTAR) model, which allows for a more flexible and heterogeneous spatial dependence structure [14]. However, the performance of these spatio-temporal models is highly dependent on the choice of spatial weight matrix used to represent inter-regional relationships.

Conventional approaches to constructing spatial weight matrices, such as contiguity and inverse distance, often fail to capture continuous and heterogeneous spatial autocorrelation. Therefore, an isotropic semivariogram offers a more adaptive alternative because it constructs spatial weights based on the actual distance between regions and the spatial variance structure of the data [6]. A spatial weight matrix in the GSTAR (1,1) model based on a semivariogram of petroleum reservoir thickness [14]. The semivariogram approach for spatial weights provides a more contextual representation of spatial dependencies in unevenly distributed regional economic phenomena. Based on this, this study applies the STARI model to monthly inflation data for the Greater Bandung area, using an isotropic semivariogram-based spatial weight matrix using population density data in the Greater Bandung area. This STARI model is limited to order 1 for autoregressive parameters and order 1 for space-time parameters. The STARI (1,1,1) model describes the dynamics of inter-regional inflation in West Java, particularly in the 5 regencies/cities of the Greater Bandung region, consisting of Bandung City, Cimahi City, Bandung Regency, West Bandung Regency, and Sumedang Regency. Parameter estimation is performed using the Ordinary Least Squares (OLS) method, while model performance evaluation is based on the level of forecasting accuracy using Mean Squared Error [11]. This approach is expected to contribute both methodologically and empirically in the development of spatio-temporal analysis of the dynamics of the regional financial system in the Greater Bandung region.

2. Research Study

2.1. **Autoregressive Model for Time Series Analysis.**

Time series analysis is a series of observations of a variable recorded sequentially at specific time intervals [16]. Within the stochastic modeling framework, some commonly used models include Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA), and Seasonal Autoregressive Integrated Moving Average (SARIMA). The Autoregressive (AR) model in univariate time series analysis aims to describe a condition where the current value of time series data depends on the value in the previous period [13]. An autoregressive (AR) model with order p or ($AR_{(p)}$) states that the current period's data is influenced by data from the previous p periods [16]. In general, it satisfies the following equation:

$$Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \cdots + \phi_p Z_{t-p} + e(t) \tag{1}$$

where,

Z_t : random variable z_t at time t;

$Z_{t-1}, \ldots, Z_{t-p}$: time series values at times $(t-1), \ldots, (t-p)$;

ϕ_i : autoregressive parameter of order i, with $i = 1, 2, \ldots, p$;

e_t : error term at time t; with $e(t) \overset{iid}{\sim} N(0, \sigma^2)$;

p : order of $AR_{(p)}$.

2.2. **Stationarity in Univariate Time Series.**

Stationarity is a crucial condition in time series analysis, where the statistical characteristics of a process remain unchanged over time. A time series is said to be stationary if it has a constant mean and variance and does not exhibit a systematic trend [7]. Thus, data fluctuations occur only around a fixed mean value without any significant upward or downward patterns [16]. The Augmented Dickey-Fuller (ADF) test is used to test data stationarity. The ADF test is an extension of the Dickey-Fuller test by adding an autoregressive component to handle autocorrelation in the data using equation (1) [16]. For a first-order AR model, the following is written:

$$Z(t) = \phi Z(t-1) + e(t), \tag{2}$$

where,

$Z(t)$: random variable at time t;

$Z(t-1)$: observation value at time $(t-1)$;

ϕ : model parameter;

$e(t)$: error term at time t.

2.3. **Space-Time Autoregressive Integrated Model.**

The Space-Time Autoregressive (STAR) model is one approach used to model time series data that has spatio-temporal dependence [12]. This model has been widely applied in various fields of science. Further development of the STAR model was carried out by Hesti through the introduction of the Space Time Autoregressive Integrated (STARI) model which is intended for non-stationary data, especially if it has a trend pattern. Because the data is not stationary in the mean, the Space Time Autoregressive Integrated (STARI) model is used. In STARI $(1, 1, 1)$ the Integrated component shows that the actual data $y(t)$ is transformed through first-order differencing ($d = 1$). Thus, the definition of the result variable of differencing 1 is expressed as $z(t)$ as follows:

$$d = 1, \qquad \mathbf{z}(t) = \mathbf{y}(t) - \mathbf{y}(t-1). \tag{3}$$

Furthermore, the STARI$(1,1,1)$ model is written in the vector form $\mathbf{z}(t)$ as follows:

$$\mathbf{z}(t) = \phi_{01}\mathbf{z}(t-1) + \phi_{11}\mathbf{W}\mathbf{z}(t-1) + \mathbf{e}(t), \tag{4}$$

with,

$\mathbf{z}(t)$: vector of random variables $\big((n(T-1)) \times 1\big)$ from n locations at time t;
$\mathbf{W}$: weight matrix $(n \times n)$ at spatial lag 1;
t : observation time $(t = 2, 3, 4, \ldots, T)$;
ϕ_{01} : model parameter at spatial lag 0 and time lag 1;
ϕ_{11} : model parameter at spatial lag 1 and time lag 1;

and the error vector,

$$\mathbf{e}(t) \overset{iid}{\sim} N(\mathbf{0}, \sigma^2\mathbf{I}).$$

The STARI$(1,1,1)$ model equation for five locations can be presented in matrix form as follows:

$$\begin{pmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \end{pmatrix} = \phi_{01} \begin{pmatrix} z_1(t-1) \\ z_2(t-1) \\ z_3(t-1) \\ z_4(t-1) \\ z_5(t-1) \end{pmatrix} + \phi_{11} \begin{pmatrix} 0 & w_{12} & w_{13} & w_{14} & w_{15} \\ w_{21} & 0 & w_{23} & w_{24} & w_{25} \\ w_{31} & w_{32} & 0 & w_{34} & w_{35} \\ w_{41} & w_{42} & w_{43} & 0 & w_{45} \\ w_{51} & w_{52} & w_{53} & w_{54} & 0 \end{pmatrix} \begin{pmatrix} z_1(t-1) \\ z_2(t-1) \\ z_3(t-1) \\ z_4(t-1) \\ z_5(t-1) \end{pmatrix} + \begin{pmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ e_4(t) \\ e_5(t) \end{pmatrix}. \tag{5}$$

In general, equation (4) for each location $i = 1, 2, 3, 4$, and 5 can be written in the form of a system of equations as follows:

$$\begin{aligned} z_1(t) &= \phi_{01}z_1(t-1) + \phi_{11}w_{12}z_2(t-1) + \phi_{11}w_{13}z_3(t-1) \\ &\quad + \phi_{11}w_{14}z_4(t-1) + \phi_{11}w_{15}z_5(t-1) + e_1(t), \\ z_2(t) &= \phi_{01}z_2(t-1) + \phi_{11}w_{21}z_1(t-1) + \phi_{11}w_{23}z_3(t-1) \\ &\quad + \phi_{11}w_{24}z_4(t-1) + \phi_{11}w_{25}z_5(t-1) + e_2(t), \\ z_3(t) &= \phi_{01}z_3(t-1) + \phi_{11}w_{31}z_1(t-1) + \phi_{11}w_{32}z_2(t-1) \\ &\quad + \phi_{11}w_{34}z_4(t-1) + \phi_{11}w_{35}z_5(t-1) + e_3(t), \\ z_4(t) &= \phi_{01}z_4(t-1) + \phi_{11}w_{41}z_1(t-1) + \phi_{11}w_{42}z_2(t-1) \\ &\quad + \phi_{11}w_{43}z_3(t-1) + \phi_{11}w_{45}z_5(t-1) + e_4(t), \\ z_5(t) &= \phi_{01}z_5(t-1) + \phi_{11}w_{51}z_1(t-1) + \phi_{11}w_{52}z_2(t-1) \\ &\quad + \phi_{11}w_{53}z_3(t-1) + \phi_{11}w_{54}z_4(t-1) + e_5(t). \end{aligned}$$

or written in matrix form in the equation below.

$$\begin{pmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \end{pmatrix}_{(5(T-1)\times 1)} = \left(\begin{pmatrix} z_1(t-1) \\ z_2(t-1) \\ z_3(t-1) \\ z_4(t-1) \\ z_5(t-1) \end{pmatrix} \begin{pmatrix} w_{12}z_2(t-1) + w_{13}z_3(t-1) + w_{14}z_4(t-1) + w_{15}z_5(t-1) \\ w_{21}z_1(t-1) + w_{23}z_3(t-1) + w_{24}z_4(t-1) + w_{25}z_5(t-1) \\ w_{31}z_1(t-1) + w_{32}z_3(t-1) + w_{34}z_4(t-1) + w_{35}z_5(t-1) \\ w_{41}z_1(t-1) + w_{42}z_2(t-1) + w_{43}z_3(t-1) + w_{45}z_5(t-1) \\ w_{51}z_1(t-1) + w_{52}z_2(t-1) + w_{53}z_3(t-1) + w_{54}z_4(t-1) \end{pmatrix} \right)_{(5(T-1)\times 1)} \begin{pmatrix} \phi_{01} \\ \phi_{11} \end{pmatrix}_{(2\times 1)} + \begin{pmatrix} e_1(t) \\ e_2(t) \\ e_3(t) \\ e_4(t) \\ e_5(t) \end{pmatrix}_{(5(T-1)\times 1)}, \tag{6}$$

$$\text{for } t = 2, 3, \ldots, T.$$

Equation (6) for location $i = 1, 2$ and time $t = 1, 2, 3$ can be written as follows:

$$\begin{pmatrix} z_1(2) \\ z_2(2) \\ z_1(3) \\ z_2(3) \end{pmatrix}_{(4\times 1)} = \left(\begin{pmatrix} z_1(1) \\ z_2(1) \\ z_1(2) \\ z_2(2) \end{pmatrix} \begin{pmatrix} w_{11}z_1(1) + w_{12}z_2(1) \\ w_{21}z_1(1) + w_{22}z_2(1) \\ w_{11}z_1(2) + w_{12}z_2(2) \\ w_{21}z_1(2) + w_{22}z_2(2) \end{pmatrix} \right)_{(4\times 2)} \begin{pmatrix} \phi_{01} \\ \phi_{11} \end{pmatrix}_{(2\times 1)} + \begin{pmatrix} e_1(2) \\ e_2(2) \\ e_1(3) \\ e_2(3) \end{pmatrix}_{(4\times 1)}.$$

Equation (6) for locations $i = 1, 2$ and times $t = 1, 2, 3$ can be written as follows:

$$\begin{pmatrix} z_1(2) \\ z_2(2) \\ z_1(3) \\ z_2(3) \end{pmatrix}_{(4\times 1)} = \begin{pmatrix} z_1(1) & w_{11}z_1(1) + w_{12}z_2(1) \\ z_2(1) & w_{21}z_1(1) + w_{22}z_2(1) \\ z_1(2) & w_{11}z_1(2) + w_{12}z_2(2) \\ z_2(2) & w_{21}z_1(2) + w_{22}z_2(2) \end{pmatrix}_{(4\times 2)} \begin{pmatrix} \phi_{01} \\ \phi_{11} \end{pmatrix}_{(2\times 1)} + \begin{pmatrix} e_1(2) \\ e_2(2) \\ e_1(3) \\ e_2(3) \end{pmatrix}_{(4\times 1)}.$$

The estimation of the parameters of the STAR(1, 1, 1) model can be carried out using the Ordinary Least Squares (OLS) method, since the model can be expressed as a linear model:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{e}(t), \qquad \mathbf{e}(t) \overset{iid}{\sim} N(\mathbf{0}, \sigma^2 I).$$

For the inflation phenomenon in the five regions of Greater Bandung, with the number of locations $i = 1, 2, 3, 4, 5$ and time $t = 1, 2, \ldots, T$, the linear form of the STARI(1, 1, 1) model is obtained as follows:

$$\mathbf{Y} = \mathbf{z}(t) = \begin{pmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \\ z_4(t) \\ z_5(t) \end{pmatrix}, \qquad \boldsymbol{\beta} = \begin{pmatrix} \phi_{01} \\ \phi_{11} \end{pmatrix},$$

and the design matrix is

$$\mathbf{X} = [\mathbf{z}(t-1) \quad \mathbf{W}\mathbf{z}(t-1)],$$

with

$$\mathbf{z}(t-1) = \begin{pmatrix} z_1(t-1) \\ z_2(t-1) \\ z_3(t-1) \\ z_4(t-1) \\ z_5(t-1) \end{pmatrix} \quad \text{and} \quad \mathbf{W}\mathbf{z}(t-1) = \begin{pmatrix} \sum_{j\neq 1} w_{1j}z_j(t-1) \\ \sum_{j\neq 2} w_{2j}z_j(t-1) \\ \sum_{j\neq 3} w_{3j}z_j(t-1) \\ \sum_{j\neq 4} w_{4j}z_j(t-1) \\ \sum_{j\neq 5} w_{5j}z_j(t-1) \end{pmatrix}. \tag{7}$$

Thus, explicitly, the matrix $\mathbf{X}$ can be written as follows:

$$\mathbf{X}_{(5(T-1))\times 2} = \begin{pmatrix} z_1(t-1) & \sum_{j\neq 1} w_{1j} z_j(t-1) \\ z_2(t-1) & \sum_{j\neq 2} w_{2j} z_j(t-1) \\ z_3(t-1) & \sum_{j\neq 3} w_{3j} z_j(t-1) \\ z_4(t-1) & \sum_{j\neq 4} w_{4j} z_j(t-1) \\ z_5(t-1) & \sum_{j\neq 5} w_{5j} z_j(t-1) \end{pmatrix}.$$

Using the obtained linear regression formulation, the parameters of the STAR$(1,1,1)$ model are estimated using the Ordinary Least Squares (OLS) method, resulting in the estimator

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}.$$

The estimated parameters of the STARI$(1,1,1)$ model are subsequently used for forecasting future observations by incorporating the influence of variables from neighboring locations around a particular location.

2.4. **Spatial Weight Matrix.** The spatial weight matrix $\mathbf{W}$ is an $(n \times n)$ matrix that represents spatial proximity among regions. The element w_{ij} indicates the influence of location j on location i, which can be formed based on regional contiguity or a certain geographical distance. Locations that are close to a particular location tend to have large w_{ij} values, whereas distant locations tend to have small w_{ij} values. The matrix $\mathbf{W}$ is given as follows:

$$\mathbf{W} = \begin{bmatrix} w_{11} & \cdots & w_{1n} \\ \vdots & \ddots & \vdots \\ w_{n1} & \cdots & w_{nn} \end{bmatrix}. \tag{8}$$

The weighting in the STAR model is determined based on the spatial distance between observation locations. The distance between locations i and j is calculated using the Euclidean distance based on latitude and longitude coordinates as follows:

$$d_{ij} = \sqrt{(u_i - u_j)^2 + (v_i - v_j)^2}.$$

The spatial weight between locations i and j is then normalized so that the sum of the weights in each row equals one, namely:

$$w_{ij} = \frac{w_{ij}}{\sum_{j=1}^{N} w_{ij}}, \qquad i \neq j.$$

2.5. **Weight Matrix using Inverse Distance.** The squared distance weighting method is a spatial weighting technique that determines the weight based on the distance between locations. The basic concept is that the closer the distance between two locations, the greater the weight of their spatial relationship, and conversely, the further the distance, the smaller the weight. Mathematically, the initial weight between location i and location j is given by [14]:

$$w_{ij} = \frac{1}{d_{ij}^2}, \qquad \sum_{j=1}^{N} w_{ij} = 1 \text{ untuk setiap } i, \tag{9}$$

where d_{ij} represents the distance between location i and location j. As an illustration, for the case of two locations arranged on a regular grid as in the matrix below, the distance matrix from the center point of each location in distance units is obtained:

$$\mathbf{W} = \begin{bmatrix} 0 & d_{12} \\ d_{21} & 0 \end{bmatrix}. \tag{10}$$

Since the distance is symmetric ($d_{12} = d_{21}$) the initial weight matrix W can be written as:

$$\mathbf{W} = \begin{bmatrix} 0 & \frac{1}{d_{12}^2} \\ \frac{1}{d_{21}^2} & 0 \end{bmatrix}. \tag{11}$$

Next, w_{ij} is the weight between locations i and j, d_{ij} is the distance between locations i and j. The squared distance weight matrix for 5 locations is:

$$\mathbf{W} = \begin{bmatrix} 0 & \frac{1}{d_{12}^2} & \frac{1}{d_{13}^2} & \frac{1}{d_{14}^2} & \frac{1}{d_{15}^2} \\ \frac{1}{d_{21}^2} & 0 & \frac{1}{d_{23}^2} & \frac{1}{d_{24}^2} & \frac{1}{d_{25}^2} \\ \frac{1}{d_{31}^2} & \frac{1}{d_{32}^2} & 0 & \frac{1}{d_{34}^2} & \frac{1}{d_{35}^2} \\ \frac{1}{d_{41}^2} & \frac{1}{d_{42}^2} & \frac{1}{d_{43}^2} & 0 & \frac{1}{d_{45}^2} \\ \frac{1}{d_{51}^2} & \frac{1}{d_{52}^2} & \frac{1}{d_{53}^2} & \frac{1}{d_{54}^2} & 0 \end{bmatrix}. \tag{12}$$

2.6. **Weight Matrix using Semivariogram.** The spatial weight matrix is an important component in spatial models, including the Space-Time Autoregressive Integrated (STARI) model, because it serves to represent the proximity or interconnectedness between locations. One approach that can be used in forming the weight matrix is the variogram method, which originates from geostatistical theory and is widely used in spatial analysis to describe the spatial dependence structure of a variable [6]. In general, a variogram is a function that describes how the difference in the value of a variable changes with the separation distance (lag distance) between two locations. The general form of the variogram function can be written as follows:

$$2\gamma(h) = \frac{1}{N(h)} \sum_{i=1}^{N(h)} \left[U(s_i) - U(s_i + h)\right]^2, \tag{13}$$

with,

$2\gamma(h)$: variogram value at distance h;

$N(h)$: number of point pairs separated by distance h;

$U(s_i)$: value of the variable at location s_i;

$U(s_i + h)$: value of the variable at a location separated by distance h from s_i.

In forming a weight matrix, the weight values w_ij can be derived from the semivariogram values $\gamma(d_ij)$ by converting them into a measure of spatial relatedness [14]. One commonly used approach is:

$$w_{ij} = \frac{1}{\gamma(h_{ij}) + 1}. \tag{14}$$

In equation (12), the denominator is added with 1 to avoid division by zero. This approach ensures that the smaller the semivariogram value (indicating close spatial relationships), the larger the weight value. Conversely, the larger the semivariogram value (weak spatial relationships), the smaller the weight value. In the context of this study, the semivariogram approach was chosen because it is more adaptive than simple weighting methods such as contiguity or distance weighting [14].

2.7. **Semivariogram Experimental.** Semivariograms are generally classified into two types, namely: Experimental semivariograms are also called cloud semivariograms and theoritical model. Semivariograms are calculated from measurement data and then plotted as a function of distance [8]. Suppose $U(s_i)$ is the measurement value at location i, while $s_h = (x_h, y_h)$ is a vector containing the spatial coordinates x, y and $h = s_1 - s_2$ is the distance between point s_1 to s_2, so that the experimental semivariogram according to equation (11) can be calculated as follows:

$$\hat{\gamma}(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} [U(x_i) - U(x_i + h)]^2, \tag{15}$$

with,
$\hat{\gamma}(h)$: experimental Semivariogram value at distance h;
h : distance between point s_1 to s_2.

In the context of spatial models, experimental semivariogram results can be utilized to form spatial weight matrices that reflect spatial relationships between locations, for example for population density data. This weight matrix not only functions as a dependency weigher in the model, but also has a direct influence on parameter estimates in spatio-temporal models such as STARI.

2.8. **Theoretical Semivariogram.** Experimental semivariograms are generally irregular in shape and difficult to interpret directly. Therefore, experimental semivariograms need to be adjusted to theoretical semivariogram models for further estimation. This study will focus on three theoretical isotropic semivariogram models: the spherical model, the Gaussian model, and the exponential model. According to [14], the general shape of these three semivariogram models can be formulated mathematically, and each has important parameters such as nugget, sill, and range, which represent the level of spatial variability, influence distance, and local fluctuations in the data from these three theoretical semivariogram models, as shown below.

$$\gamma_s(h) = \begin{cases} C\left(\frac{3}{2} \cdot \frac{|h|}{a} - \left(\frac{1}{2} \cdot \frac{|h|^3}{a^3}\right)\right) & , h \leq a \\ C & , h > a \end{cases} \tag{16}$$

$$\gamma_g(h) = \begin{cases} C\left(1 - \exp\left(-\frac{h^2}{a^2}\right)\right) & , h \leq a \\ C & , h > a \end{cases} \tag{17}$$

$$\gamma_e(h) = \begin{cases} C\left(1 - \exp\left(-\frac{h^2}{a^2}\right)\right) & , h \leq a \\ C & , h > a \end{cases} \tag{18}$$

with,
$\gamma_s(h)$: Theoretical Spherical Semivariogram model
$\gamma_g(h)$: Theoretical Gaussian Semivariogram model
$\gamma_e(h)$: Theoretical Exponential Semivariogram model
h : Distance between sample locations
C : *Sill* (Variance).

According to [14], the experimental semivariogram is obtained by calculating the estimated semivariogram values γ_h for various lag distances h based on observational data. These values are then used to construct a spatial correlation function that will form the basis for weighting. The general steps in determining the weight matrix from the experimental semivariogram include:

(1) Calculate the distance between locations
(2) Estimate the semivariogram value for each pair of locations
(3) Convert the semivariogram values to spatial weights
(4) Construct the initial weight matrix.
(5) Row-standardization.

The spatial weight matrix $\mathbf{W}$ with $d_{ij} = h_{ij}$ standardized for 5 locations is expressed as follows,

$$\mathbf{W}^* = \begin{pmatrix} 0 & w_s(h_{12}) & w_s(h_{13}) & w_s(h_{14}) & w_s(h_{15}) \\ w_s(h_{21}) & 0 & w_s(h_{23}) & w_s(h_{24}) & w_s(h_{25}) \\ w_s(h_{31}) & w_s(h_{32}) & 0 & w_s(h_{34}) & w_s(h_{35}) \\ w_s(h_{41}) & w_s(h_{42}) & w_s(h_{43}) & 0 & w_s(h_{45}) \\ w_s(h_{51}) & w_s(h_{52}) & w_s(h_{53}) & w_s(h_{54}) & 0 \end{pmatrix}$$

2.9. **Selection of the Best STARI (1,1,1) Model Using MSE.** The selection of the best model in STARI (1,1,1) is done by evaluating the model's ability to produce accurate forecasts. One measure commonly used to assess forecasting performance is the Mean Squared Error (MSE), because it is able to represent the average squared difference between the actual value and the estimated or forecasted value. Mathematically, the MSE for all location is defined as [11]:

$$MSE = \frac{1}{nT}\sum_{i=1}^{n}\sum_{t=1}^{T}((z_i(t) - \hat{z}_i(t))^2 \tag{19}$$

where $z_i(t)$ is the actual observation value at time t, $\hat{z}_i(t)$ is the estimated value of the STARI$(1,1,1)$ model at the i-th location, and T represents the number of observations in time. In this study, the MSE value is calculated for each STARI$(1,1,1)$ model formed based on a combination of the spatial weight matrix structure and the estimated parameters. The STARI$(1,1,1)$ model with the smallest MSE value is selected as the best model, because it shows the lowest level of forecasting error and better model ability to capture spatial and temporal dependencies simultaneously in the analyzed data.

3. RESULTS AND DISCUSSION

3.1. **Descriptive Statistics of Inflation Data in the Five Regions of Greater Bandung.** This study uses monthly inflation data in five regions of Greater Bandung as spatio-temporal observation variables from January 2020 to December 2025 [3]. For the weight matrix, a uniform weight matrix based on neighborhood, a squared distance matrix, and a semivariogram weight matrix were selected using population density data in the Greater Bandung region from the Central Bureau of Statistics (2021). Descriptive statistics are used to provide an initial overview of the characteristics of inflation data in each observation region in the Greater Bandung region. Measures of central tendency and dispersion of inflation data are presented to identify differences in inflation rates between regions and their levels of variation during the observation period. Descriptive statistics regarding measures of central tendency and dispersion of inflation data are presented in Table 1. Table 1 shows that Sumedang Regency has the highest average inflation rate, while Bandung City has the lowest average inflation rate. Cimahi City, Bandung Regency, and West Bandung Regency have relatively close average inflation rates. In terms of distribution, the standard deviation of inflation across all regions is relatively uniform. This indicates that inflation data variation across regions is not significantly different. Next, a time series plot was created to illustrate the monthly inflation phenomenon in

TABLE 1. Descriptive Statistics of Monthly Inflation Data in the Greater Bandung Region

Region	**Minimum**	**Maximum**	**Mean**	**Standard Deviation**
Bandung City	0.0463	0.1009	0.0748	0.0171
Cimahi City	0.0475	0.1054	0.0763	0.0175
Bandung Regency	0.0461	0.1045	0.0770	0.0174
West Bandung Regency	0.0472	0.1092	0.0779	0.0179
Sumedang Regency	0.0487	0.1079	0.0788	0.0179

each region. This time series plot was used to observe the movement patterns and fluctuations in inflation over time.

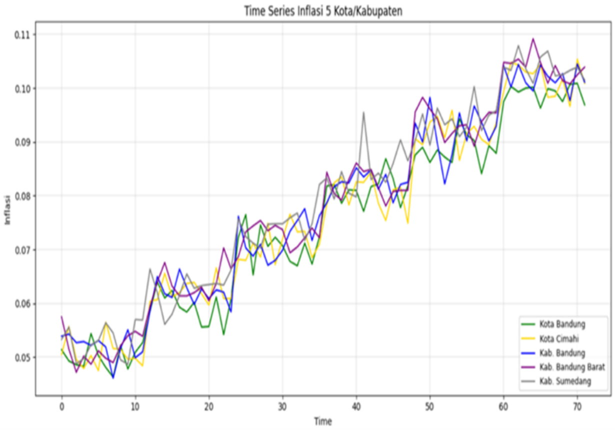

FIGURE 1. Time series plot of inflation data in five regions of Greater Bandung

Based on the time series plot, it can be seen that inflation in the five cities/regencies fluctuated and exhibited a trend pattern over time. Although there were differences in inflation rates between regions, the fluctuation patterns within each region showed a relatively similar trend. This is evident from the upward and downward movements in inflation over a nearly simultaneous period across all observation areas. This similarity in patterns indicates a temporal and spatial interconnectedness of inflation dynamics across regions, such that changes in inflation in one region tend to be followed by changes in other regions.

3.2. **Stationary Test of Inflation Data.** Univariate identification of stationarity in inflation time series data was performed to ensure that the data met the basic assumptions in Space-Time Autoregressive Integrated (STARI) modeling. Stationarity in the mean was tested using the Augmented Dickey-Fuller (ADF) test. The ADF test was performed before and after the differencing process to determine the presence of a unit root in each observation area [16]. The results of the ADF test on inflation data before and after differencing are presented in Table 2.

TABLE 2. Augmented Dickey Fuller (ADF) Test of Inflation Data in the Greater Bandung Region

Region	**Variable**	*p-value* before *differencing*	**Conclusion**	*p-value* after *differencing*	**Conclusion**
Bandung City	$Z_1(t)$	0.82079	Non-stationary Data	0.00000	Stationary Data
Cimahi City	$Z_2(t)$	0.89819	Non-stationary Data	0.00000	Stationary Data
Bandung Regency	$Z_3(t)$	0.81304	Non-stationary Data	0.00000	Stationary Data
West Bandung Regency	$Z_4(t)$	0.82590	Non-stationary Data	0.00000	Stationary Data
Sumedang Regency	$Z_5(t)$	0.79656	Non-stationary Data	0.00001	Stationary Data

Based on Table 2, it is known that all inflation data in the five regions before differencing had a p-value greater than the significance level of $\alpha = 5\%$. This indicates that the inflation data in all regions are not stationary in the mean and still contain a unit root. Next, a first-order differencing process was performed to eliminate non-stationarity in the data. The results of the ADF test after differencing showed that all p-values were smaller than $\alpha = 5\%$. Thus, the null hypothesis stating the presence of a unit root is rejected, and it can be concluded that the inflation data resulting from differencing is stationary in the mean.

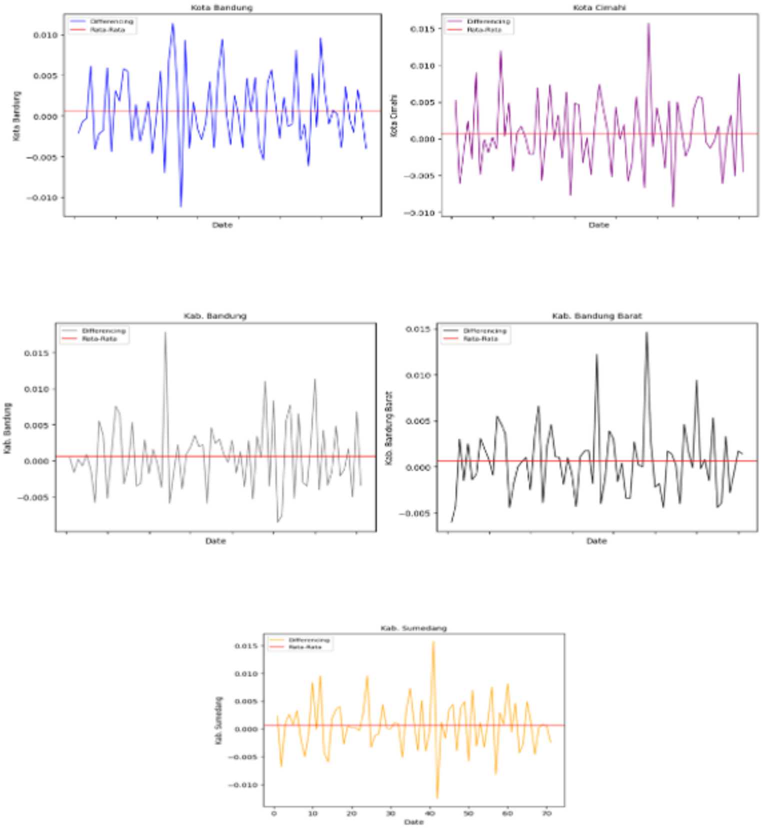

Figure 2. Plot of differencing inflation data in five regions of Greater Bandung

To support the ADF test results, visual analysis was performed using Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots on the differencing inflation data for each region. Based on the ACF and PACF plots, it can be seen that most of the autocorrelation values are around the average value of zero. Thus, it can be concluded that the inflation data after first-order differencing meets the stationarity assumption and is suitable for use in Space-Time Autoregressive Integrated (STARI) modeling in the next analysis stage.

3.3. Identification of the Order of the Univariate Time-Series Model.

Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) plots are used as tools in the model identification process to determine the optimal time lag for inflation data. This analysis was performed on differencing inflation data for each city/district to identify any remaining temporal dependency patterns after the data was made stationary. The results, plotted in Figure 3, are presented.

FIGURE 3. Plot of differencing inflation data in five regions of Greater Bandung

The ACF and PACF plots presented in Figure 3 show that the autocorrelation pattern in the differencing inflation data for all cities/regencies is cut off at lag one. Because the inflation data has undergone a one-time differencing process to achieve stationarity, the univariate model formed is ARI(1,1). Furthermore, considering the inter-regional interrelationships, a spatial lag of one is used to represent the spatial relationship between Bandung City, Cimahi City, Bandung Regency, West Bandung Regency, and Sumedang Regency as a single observation area. Thus, the model used in this study is the Space Time Autoregressive Integrated STARI(1,1,1) model, order 1 for autoregressive, differencing d=1, and the location position is in spatial lag 1 or one group.

3.4. **Weight Matrix of STARI (1,1,1) Model.** This study uses three approaches to weight matrix formation: a uniform weight matrix, an inverse distance weight matrix, and a weight matrix based on semivariogram. All weight matrices have been row-normalized, so that the sum of the weights in each row is equal to one. The formation of the weight matrices is based on the spatial configuration of the five observation areas as shown in the regional map in Figure 4, which represents the geographic proximity between areas in the analyzed forecasting system.

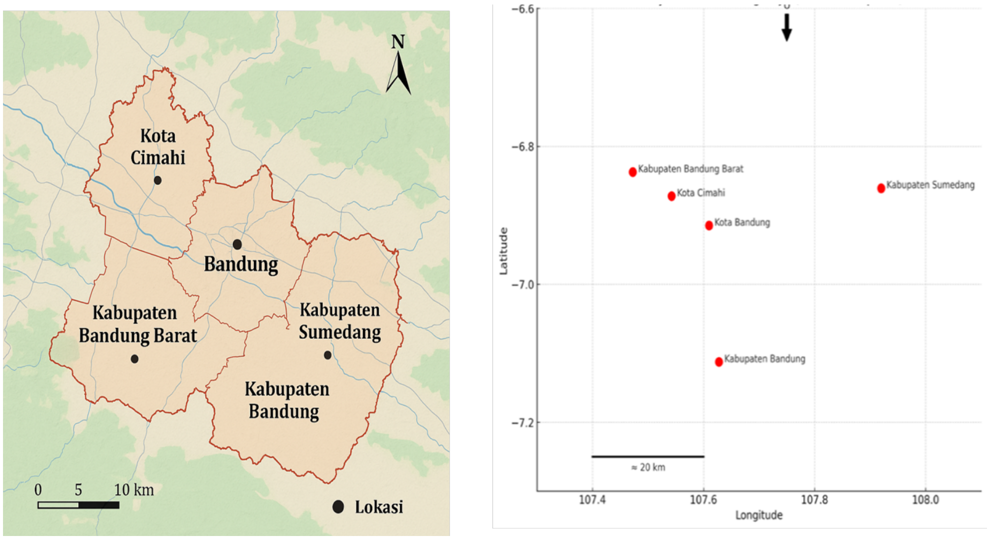

FIGURE 4. Location Map in the five regions of Greater Bandung

3.4.1. *Uniform Weight Matrix.* A uniform weight matrix is formed based on the neighborhood relationships between regions in the form of a binary matrix, then normalized row by row. The resulting uniform weight matrix is as follows:

$$\mathbf{W}_{\text{uniform}} = \begin{pmatrix} 0 & 0.5 & 0.5 & 0 & 0 \\ 0.3333 & 0 & 0.3333 & 0.3333 & 0 \\ 0.25 & 0.25 & 0 & 0.25 & 0.25 \\ 0 & 0.3333 & 0.3333 & 0 & 0.3333 \\ 0 & 0 & 0.5 & 0.5 & 0 \end{pmatrix}.$$

This matrix assumes that each region is influenced equally by its neighboring regions, without considering geographic distance or spatial autocorrelation structure.

3.4.2. *Weight Matrix using Inverse Distance for Greater Bandung Region.* The co-distance weight matrix is formed based on the geographic distance between regions using the co-distance squared function, namely $w_{ij} = \frac{1}{d_{ij}^2}$ for $i \neq j$. After row normalization, the weight matrix is obtained as follows:

$$\mathbf{W}_{ID} = \begin{pmatrix} 0 & 0.563473 & 0.162690 & 0.245132 & 0.028705 \\ 0.228587 & 0 & 0.048159 & 0.714156 & 0.009098 \\ 0.400040 & 0.291902 & 0 & 0.270247 & 0.037812 \\ 0.114884 & 0.825034 & 0.051508 & 0 & 0.008574 \\ 0.338484 & 0.264457 & 0.181324 & 0.215735 & 0 \end{pmatrix}.$$

This weight matrix shows that the greatest spatial influence comes from areas with relatively closer geographical distances.

3.4.3. *Weight matrix using Semivariogram for Greater Bandung Region.* The semivariogram approach was used to capture the continuous and heterogeneous spatial autocorrelation structure of population density data in the Greater Bandung area. Spatial weights were formed using the semivariogram value, i.e., $w_{ij} = \frac{1}{\gamma(h_{ij})+1}$, and then row-normalized [14]. Population density data from five districts/cities in the Greater Bandung area were selected for the semivariogram, as population density is related to inflation in these areas.

Initially, an empirical semivariogram of the population density data in the Greater Bandung area was constructed based on inter-regional distances. It was then matched with three theoretical semivariogram models: spherical, exponential, and Gaussian. The three models were compared to determine the best semivariogram model to be used in the formation of the spatial weight matrix for the STARI(1,1,1) model. A visualization of the theoretical semivariogram curves for each model is presented in Figure 5.

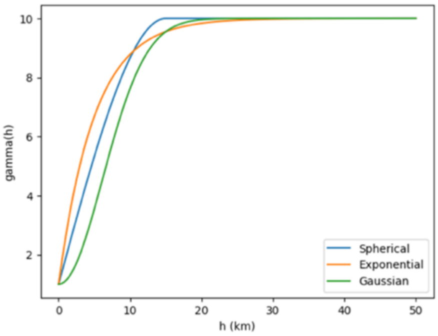

FIGURE 5. Plot of Three Theoretical Semivariogram Models

To obtain the best experimental semivariogram estimate, the MSE and Sum Squared Error (SSE) were calculated by comparing the error between the experimental semivariogram and the theoretical semivariogram model. The results of the MSE and SSE calculations, as well as the sill and range of the three semivariogram models for population density data in the Greater Bandung area, are presented in Table 3. Based on Table 3, the best experimental semivariogram estimate for population density data is the exponential semivariogram model. The exponential semivariogram weight matrix obtained for population density data in the five regions of Greater Bandung is as follows:

TABLE 3. Semivariogram Model of Population Density Data in the Greater Bandung Area

Semivariogram Model	**MSE**	**SSE**	*Nugget*	*Sill*	*Range* (km)
Exponential	**0.01185**	**0.03554**	0.12938	0.46881	22.31
Spherical	0.01364	0.04092	0.12938	0.46881	22.31
Gaussian	0.01950	0.05849	0.12938	0.48099	22.31

$$W_{semivariogram} = \begin{pmatrix} 0 & 0.279267 & 0.242947 & 0.250584 & 0.0227202 \\ 0.254274 & 0 & 0.219042 & 0.320894 & 0.205790 \\ 0.257619 & 0.255101 & 0 & 0.248019 & 0.239261 \\ 0.235794 & 0.331633 & 0.220089 & 0 & 0.212484 \\ 0.251144 & 0.249835 & 0.249412 & 0.249609 & 0 \end{pmatrix}.$$

This weight matrix reflects a more even spatial relationship but remains heterogeneous between regions, as it is formed based on the spatial autocorrelation structure of population density data. Furthermore, this study uses three weight matrices: uniform, inverse distance, and based on the exponential semivariogram of population density data in the STARI(1,1,1) model. To obtain the best STARI(1,1,1) model, a model comparison criterion using MSE was used.

3.4.4. *Selection of the Best STARI (1,1,1) Model using MSE for Financial Data at Greater Bandung.* Furthermore, the estimated STARI(1,1,1) model with a spatial weight matrix in the form of a uniform, semi-distance matrix and based on a semivariogram was selected for the best model using the SSE and MSE value criteria presented in Table 4.

Table 4. Parameter Estimation and Error Values of the STARI(1,1,1) Model Based on Weight Matrices

Weight Matrix	ϕ_{01} **(time parameter)**	ϕ_{11} **(space-time parameter)**	**SSE**	**MSE**
Uniform	−0.3206	0.2241	0.00645	0.00001843
Inverse distance	−0.3206	0.2241	0.00644	0.00429
Semivariogram based	−0.3467	0.3274	**0.00635**	**0.00001814**

Based on Table 4, the STARI(1,1,1) model with a semivariogram-based for matrix using population density data yielded the smallest SSE and MSE values compared to the uniform weight matrix and the inverse distance weight matrix. This indicates that the weight matrix using semivariogram for population density data is able to better represent spatial dependency in modeling inter-regional inflation in the five regions of Greater Bandung.

Estimation of inflation data using the STARI(1,1,1) model with the semivariogram weight matrix for population density data for each location can be described as follows:

$$\begin{aligned}
\text{Bandung City:}\quad \hat{z}_1(t) &= -0.3467\, z_1(t-1) + 0.3274\big(0.2793\, z_2(t-1) + 0.2429\, z_3(t-1) \\
&\quad + 0.2506\, z_4(t-1) + 0.2272\, z_5(t-1)\big) \\
\text{Cimahi City:}\quad \hat{z}_2(t) &= -0.3467\, z_2(t-1) + 0.3274\big(0.2543\, z_1(t-1) + 0.2190\, z_2(t-1) \\
&\quad + 0.3209\, z_4(t-1) + 0.2058\, z_5(t-1)\big) \\
\text{Bandung Regency:}\quad \hat{z}_3(t) &= -0.3467\, z_3(t-1) + 0.3274\big(0.2576\, z_1(t-1) + 0.2551\, z_2(t-1) \\
&\quad + 0.2480\, z_4(t-1) + 0.2393\, z_5(t-1)\big) \\
\text{West Bandung Regency:}\quad \hat{z}_4(t) &= -0.3467\, z_4(t-1) + 0.3274\big(0.2358\, z_1(t-1) + 0.3316\, z_2(t-1) \\
&\quad + 0.2201\, z_3(t-1) + 0.2125\, z_5(t-1)\big) \\
\text{Sumedang Regency:}\quad \hat{z}_5(t) &= -0.3467\, z_5(t-1) + 0.3274\big(0.2511\, z_1(t-1) + 0.2494\, z_2(t-1) \\
&\quad + 0.2494\, z_3(t-1) + 0.2496\, z_4(t-1)\big)
\end{aligned}$$

For example, for Sumedang Regency, it can be stated that inflation at time t is influenced by inflation in Sumedang Regency one time ago of −34.67%, plus the influence of inflation from Bandung City of 8.22% plus inflation in Cimahi City of 8.17%, inflation in Bandung Regency of 8.17% and inflation in West Bandung Regency of 8.18%. By using a weight matrix based on the semivariogram of population density data, it can be explained that inflation in Sumedang Regency at time $(t-1)$ reduces inflation by 34.67%. While the influence of inflation in other regencies/cities in the Greater Bandung area increases inflation in Sumedang Regency by around 8.2%. This shows the existence of interactions between locations to describe variations in inflation data in a particular location and the influence of inflation in other locations through a weight matrix using population density data.

3.4.5. *Inflation Data Forecasting using the STARI (1,1,1) Model with a Weight Matrix based on Semivariongam.*

The STARI (1,1,1) model with semivariogram-based spatial weights is the best model for forecasting inflation in five regions in Greater Bandung. Based on the parameter estimation results, the forecasting model for each location in the Greater Bandung region for January to March 2026 is as follows.

Table 5. Forecasting of Inflation Data Using the STARI $(1,1,1)$ Model

Time	**Bandung City**	**Cimahi City**	**Bandung Regency**	**West Bandung Regency**	**Sumedang Regency**
January 2026	1.766748	1.766748	1.766748	1.766748	1.766748
February 2026	1.740062	1.740062	1.740062	1.740062	1.740062
March 2026	1.742635	1.742635	1.742635	1.742635	1.742635

Based on forecasting results using the STARI (1,1,1) model with a semivariogram-based spatial weight matrix, the inflation forecast for the period from January to March 2026 was relatively stable across all regions in Greater Bandung. The forecasted inflation value in January 2026 was recorded at 1.766748, then decreased slightly in February 2026 to 1.740062, and then increased again in March 2026 to 1.742635. This pattern indicates moderate inflation fluctuations and no extreme spikes in the short term.

The uniformity of forecast values across regions indicates that inflation dynamics in the five regions of Greater Bandung are strongly spatially linked, so that changes in inflation in one region tend to be followed by changes in other regions. This aligns with the characteristics of the STARI model, which simultaneously accommodates time and spatial dependencies, and the use of a weight matrix based on semivariogram, which is capable of capturing spatial correlation structures based on geographic proximity and interregional variation. Thus, this model is considered suitable for use as a tool for short-term inflation forecasting in the Greater Bandung area, especially to support the formulation of regional economic policies based on data and inter-regional linkages in the Greater Bandung area.

4. CONCLUSION AND SUGGESTIONS

4.1. **Conclusion.** Based on the results of the analysis and discussion that have been carried out, the following conclusions can be drawn:

(1) The spatial weight matrix based on the semivariogram describes spatial autocorrelation according to the characteristics of each location in forecasting inflation in the Greater Bandung area.
(2) The STARI $(1,1,1)$ model was constructed using monthly inflation data that had undergone first-order differencing and satisfied the basic assumptions of simultaneous spatio-temporal modeling.
(3) Based on the accuracy criteria, the STARI $(1,1,1)$ model with semivariogram weights produced the smallest MSE value of 0.00001814 compared to the model using uniform weight matrices and inverse-distance weight matrices. Therefore, the model can be used for short-term forecasting within the next 1–3 months.

4.2. **Suggestion.** Based on the research results obtained, several suggestions for future studies are as follows:

(1) Future research may develop models with higher temporal and spatial orders to improve the accuracy of medium-term and long-term forecasting.
(2) The use of other spatial weight matrices, such as anisotropic semivariograms or the inclusion of exogenous variables, may be considered to broaden the application of spatio-temporal models.

Acknowledgement

The author would like to thank the reviewers who have dedicated their time, effort, and thought to providing constructive assessments, guidance, and feedback from the planning stage through the completion of this scientific article. Their contributions have been invaluable in improving the quality of the substance and presentation of this article.

References

[1] Abanyam, N. L., and Angahar, P. A. (2023). Fiscal expansion and public debt sustainability in developing economies. *Journal of Economic Policy*, 18(2), 145–162.

[2] Badan Pusat Statistik. (2021). *Perhitungan Indeks Harga Konsumen dan Inflasi.*, Jakarta: BPS RI.

[3] Bank Indonesia. (2023). *Laporan Perekonomian Indonesia Tahun 2023. Jakarta: Bank Indonesia.*, Diakses dari https://www.bi.go.id.

[4] Boediono. (2013). Ekonomi Moneter. Yogyakarta: BPFE.

[5] Chen, X., Liu, Y., and Zhang, H. (2023). Spatial heterogeneity and regional economic dynamics. *Spatial heterogeneity and regional economic dynamics. Regional Science and Urban Economics*, 98, 103825.

[6] Cressie, N. (1993). *Statistics for Spatial Data*, New York: John Wiley and Sons.

[7] Cryer, J. D. (1986). *Time Series Analysis.* Boston: PWS-Kent Publishing Company.

[8] Gaetan, C., and Guyon, X. (2010). *Spatial Statistics and Modeling.* New York: Springer.

[9] Glocker, C., Giesen, S., and Tillmann, P. (2023). Spatial dependence and inflation: Evidence from European regions. *Regional Studies.* 57(2), 173–188.

[10] Lawrence, M., Goodwin, P., O'Connor, M., and Önkal, D. (2009). *Judgmental forecasting: A review of progress over the last 25 years. International Journal of Forecasting.*. 22(3), 493–518.

[11] Nasution, M. D. T., and Prasetyawan, Y. (2008). Evaluasi akurasi model peramalan menggunakan metode Mean Square Error (MSE). *Jurnal Sistem dan Manajemen Industri.* 2(1), 22–28.

[12] Pfeifer, P. E., and Deutsch, S. J. (1980). A three-stage iterative procedure for space–time modeling. *Journal of the American Statistical Association.* 75(369), 164–173.

[13] Rosyidah, E., dkk. (2017). Analisis model autoregressive pada data deret waktu. *Jurnal Statistika.* 5(2), 101–110.

[14] Ruchjana, B. N. (2002). Suatu Model Generalized Space-Time Autoregressive dan Penerapannya pada Data Produksi Minyak Bumi. Disertasi tidak dipublikasikan. Bandung: Institut Teknologi Bandung.

[15] Sagala, R., and Wibowo, D. (2021). Determinan inflasi di Indonesia menggunakan pendekatan geographically weighted regression.*Jurnal Ekonomi dan Pembangunan Indonesia.* 21(1), 88–105.

[16] Wei, W. W. S. (2019). Time Series Analysis: Univariate and Multivariate Methods (2nd ed.). Boston: Pearson/Addison Wesley.

[17] Yasin, H. (2020). Evaluasi performa model ARIMA dan VAR terhadap prediksi inflasi Indonesia. *Jurnal Ekonomi dan Statistik Indonesia.* 6(1), 24–31.