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Alternative Proofs for the Side Trisector Lengths Theorem of a Triangle

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Abstract
Discussions of side trisectors are generally more limited than those on angle trisectors. For angle trisectors, most research focuses on Morley's Theorem and its extensions. Meanwhile, studies on side trisectors usually calculate lengths using the triangle's sides and angles. A common problem is to find the length of a side trisector from its opposite vertex when only the side lengths are known. If the side trisector is extended to form a tangential excircle, can we determine its radius? This article presents several alternative proofs of the lengths produced by the side trisectors in a triangle. The main focus is to derive a formula for the trisector's length using only the triangle's original side lengths and to find the radius of the tangential excircle. These proofs employ geometric approaches such as trigonometry, Stewart's Theorem, and the Pythagorean Theorem. The result is a standard formula for the trisector length, which is then used to find the radius of the triangle's tangential excircle.

Keywords: *Tangential Excircle, Side Trisector, Stewart's Theorem*

1. Introduction

In geometry, triangles have various special lines, including the altitude, median, and angle bisector [1]. The angle bisector features many standard formulas and alternative proofs [2, 4, 5, 6]. Angle trisection, which divides an angle into three equal parts, has also been studied extensively. For example, Morley's Theorem states that angle trisectors will always form an equilateral triangle [7, 8]. Previous studies have discussed the side lengths and area ratios produced by angle trisectors [9, 10]. This work includes the concept of the mixtilinear excircle, which appears in triangles formed by these trisectors [11].

Furthermore, side trisectors are less developed than angle trisectors. Early research shows it is possible to divide a line into three equal parts if the lines are parallel [12]. However, a formula for the resulting side lengths is not widely discussed. In [13], the author examined changes to a triangle's shape when its longest side is repeatedly divided into three equal parts. That paper did not give a formula for the resulting side lengths. Similarly, [14] gave only a

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visual discussion of side trisectors. References [15] and [16] focused on symmedian lines and modifying triangle areas using Varignon's Theorem. Until now, no explicit discussion of a side-length formula for side trisectors has been found.

Based on these conditions, this paper aims to derive a formula for the side lengths of a triangle's side trisectors. The novelty is to give an explicit formula for the trisector's length using only the triangle's original side lengths. To confirm originality, the proofs use several distinct methods: the law of sines, the law of cosines, Stewart's Theorem, and the Pythagorean Theorem. The side trisector formula is then used to find the radius of the tangential excircle in the triangle formed by these trisectors.

2. Some Concepts

The following definitions and theorems will support the discussion in this study. They serve as the basis for deriving the formula for the length of the side trisector.

2.1. **Side Trisector.** In geometry, a trisector is defined as a line, point, or curve that divides an object, such as a line segment, angle, or figure, into three equal parts [17, 18].

Definition 2.1. [12, 15, 19, 20]. *The side trisectors of a triangle are two points on a side of the triangle that divide it into three parts of equal length.*

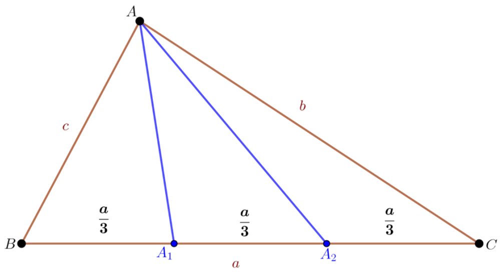

Figure 1. Triangle ABC with trisector points on a side BC

In a triangle ABC, the lines AA_1 and AA_2 are given on a side BC such that $BA_1 = A_1A_2 = A_2C = \frac{a}{3}$.

2.2. **Stewart's Theorem.** This theorem uses a line that divides a triangle's sides in a certain ratio.

Theorem 2.2. [2, 3]. *Given a triangle ABC, let point X be on side BC such that $BC : XC = r : s$. If the length of side AX is p, then the following holds:*

$$a(p^2 + rs) = b^2 r + c^2 s.$$

2.3. **Tangential Excircle.** The concept of triangle tangent circles involves circles that touch the sides of a triangle. These can be either internal or external. The definitionn of the tangential excircle of a triangle is as follows [2].

Definition 2.3. [2]. *A tangential excircle of a triangle is a circle that touches exactly one side of the triangle and the extensions, which are the straight lines formed by continuing the other two sides beyond their vertices.*

In [2], a theorem regarding the radius of the tangential excircle in a triangle is also provided.

Theorem 2.4. [2]. *In triangle ABC, let $BC = a$, $AC = b$, and $AB = c$ with $\angle ABC = \beta$. The radius of the tangential excircle on a side AC is given by:*

$$R_b = s \tan \tfrac{1}{2}\beta$$

3. Result and Discussion

This paper derives formulas for the side lengths of the triangle's side trisectors using only the original side lengths. To achieve this, several alternative proofs are used, specifically the law of sines, the law of cosines, Stewart's Theorem, and the Pythagorean Theorem. Furthermore, the resulting formulas are applied to derive the radius of the tangential excircle of the triangle constructed from these side trisectors.

Theorem 3.1. *In triangle ABC, let the side trisectors be constructed such that BB_1 and BB_2 divide side b. Given $BC = a$, $AC = b$, $AB = c$, $\angle BAC = \alpha$, $\angle ABC = \beta$, $\angle ACB = \gamma$, $\angle B_1BB_2 = \theta$, $\angle BB_1B_2 = \delta$, $\angle B_1B_2B = \tau$, then the lengths of BB_1 and BB_2 are:*

$$BB_1 = \frac{1}{3}\sqrt{3a^2 - 2b^2 + 6c^2}$$

$$BB_2 = \frac{1}{3}\sqrt{6a^2 - 2b^2 + 3c^2}$$

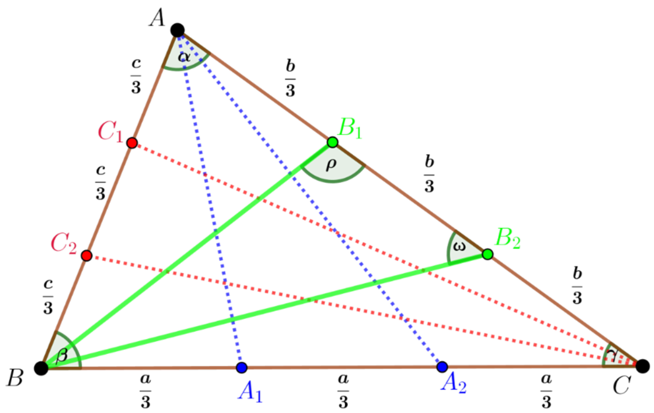

Figure 2. Trisectors of each side in triangle ABC

Proof. Theorem 3.1 can be illustrated in Figure 2. Since the side AC is divided by trisectors such that $AB_1 = B_1B_2 = B_2C = \frac{b}{3}$, then $AB_2 = B_1C = \frac{2b}{3}$.

To determine the lengths of BB_1 and BB_2, we can use four different methods: the law of cosines, the law of sines, Stewart's Theorem, and the Pythagorean Theorem.

Method 1. Using the Law of Cosines

Consider triangle $\triangle ABB_1$. Applying the law of cosines, we obtain:

$$BB_1^2 = AB^2 + AB_1^2 - 2 \cdot AB \cdot AB_1 \cdot \cos\alpha \tag{1}$$

By substituting $\cos\alpha = \dfrac{b^2 + c^2 - a^2}{2bc}$, equation (1) becomes:

$$BB_1^2 = c^2 + \left(\frac{b}{3}\right)^2 - 2 \cdot c \cdot \frac{b}{3} \cdot \frac{b^2 + c^2 - a^2}{2bc}$$

$$BB_1^2 = c^2 + \frac{b^2}{9} - \frac{b^2 + c^2 - a^2}{3} = \frac{3a^2 + 6c^2 - 2b^2}{9}$$

$$BB_1^2 = \frac{1}{9}(3a^2 + 6c^2 - 2b^2)$$

Thus,

$$BB_1 = \frac{1}{3}\sqrt{3a^2 - 2b^2 + 6c^2}. \tag{2}$$

Similarly, using the same method as in (2) for triangle $\triangle ABB_2$, we also obtain:

$$BB_2 = \frac{1}{3}\sqrt{6a^2 - 2b^2 + 3c^2}. \tag{3}$$

By following the same steps as in (2) and (3), we find that:

- For the trisector points on the side AB, the lengths of CC_1 and CC_2 satisfy:

$$CC_1 = \frac{1}{3}\sqrt{3a^2 + 6b^2 - 2c^2}$$

$$CC_2 = \frac{1}{3}\sqrt{6a^2 + 3b^2 - 2c^2}$$

- For the trisector points on the side BC, the lengths of AA_1 and AA_2 satisfy:

$$AA_1 = \frac{1}{3}\sqrt{3b^2 + 6c^2 - 2a^2}$$

$$AA_2 = \frac{1}{3}\sqrt{6b^2 + 3c^2 - 2a^2}$$

Method 2. Using the Law of Sines

The sum of angles in a triangle is 180°, then $\angle BB_2C = \delta = 180^\circ - (\theta + \gamma)$. Applying the law of sines to $\triangle BCB_1$ gives:

$$\frac{BB_1}{\sin\angle BCB_1} = \frac{B_1C}{\sin\angle B_1BC} = \frac{BC}{\sin\angle BB_1C}$$

So that,

$$\frac{BB_1}{\sin\gamma} = \frac{2b/3}{\sin\theta} = \frac{a}{\sin(180^\circ - \theta + \gamma)} \tag{4}$$

From equation (4), we obtain:

$$\begin{aligned}
\frac{2b/3}{\sin\theta} &= \frac{a}{\sin(\theta+\gamma)} \\
\frac{\sin(\theta+\gamma)}{\sin\theta} &= \frac{3a}{2b} \\
\frac{\sin\theta\cos\gamma + \cos\theta\sin\gamma}{\sin\theta} &= \frac{3a}{2b} \\
\cos\gamma + \cot\theta\cdot\sin\gamma &= \frac{3a}{2b} \\
\cot\theta &= \frac{\frac{3a}{2b} - \cos\gamma}{\sin\gamma}
\end{aligned}$$

$$\cot\theta = \frac{\frac{3a}{2b} - \cos\gamma}{\sin\gamma} \tag{5}$$

By using the trigonometric identity $\sin\theta = \dfrac{1}{\sqrt{\cot^2\theta + 1}}$ and substituting the value of $\cot\theta$, we get:

$$\sin\theta = \frac{1}{\sqrt{1 + \left(\dfrac{\frac{3a}{2b} - \cos\gamma}{\sin\gamma}\right)^2}} = \frac{1}{\dfrac{\sqrt{\sin^2\gamma + \left(\dfrac{3a}{2b} - \cos\gamma\right)^2}}{\sin\gamma}}$$

$$\sin\theta = \frac{\sin\gamma}{\sqrt{\sin^2\gamma + \left(\dfrac{3a}{2b} - \cos\gamma\right)^2}} \tag{6}$$

Next, from equation (4):

$$\frac{BB_1}{\sin\gamma} = \frac{\frac{2b}{3}}{\sin\theta}$$

So that,

$$BB_1 = \frac{2b}{3} \cdot \frac{\sin\gamma}{\sin\theta} \tag{7}$$

Substituting (6) into equation (7) yields:

$$BB_1 = \frac{2b}{3} \cdot \frac{\sin\gamma}{\frac{\sin\gamma}{\sqrt{\sin^2\gamma + \left(\frac{3a}{2b} - \cos\gamma\right)^2}}} = \frac{2b}{3} \cdot \sqrt{\sin^2\gamma + \left(\frac{3a}{2b} - \cos\gamma\right)^2}$$

$$BB_1 = \frac{2b}{3} \cdot \sqrt{\sin^2\gamma + \cos^2\gamma + \frac{9a^2}{4b^2} - \frac{3a}{b} \cdot \cos\gamma}$$

Using the identity $\sin^2\gamma + \cos^2\gamma = 1$, we have:

$$BB_1 = \frac{2b}{3} \cdot \sqrt{1 + \frac{9a^2}{4b^2} - \frac{3a}{b} \cdot \cos\gamma} \tag{8}$$

By squaring both sides and substituting $\cos\gamma = \dfrac{a^2 + b^2 - c^2}{2ab}$ into equation (8), it becomes:

$$BB_1^2 = \frac{4b^2}{9} + a^2 - \frac{4ab}{3}\left(\frac{a^2 + b^2 - c^2}{2ab}\right)$$

$$BB_1^2 = \frac{4b^2}{9} + a^2 - \frac{2a^2}{3} - \frac{2b^2}{3} + \frac{2c^2}{3}$$

$$BB_1^2 = \frac{3a^2 - 2b^2 + 6c^2}{9}$$

Thus,

$$BB_1 = \frac{1}{3}\sqrt{3a^2 - 2b^2 + 6c^2}. \tag{9}$$

Similarly, using the same steps as in (9) for $\triangle ABCB_1$, we also obtain:

$$BB_2 = \frac{1}{3}\sqrt{6a^2 - 2b^2 + 3c^2}. \tag{10}$$

By following the same steps as in (9) and (10), we find that:

- For the trisector points on the side AB, the lengths of CC_1 and CC_2 satisfy:

$$CC_1 = \frac{1}{3}\sqrt{3a^2 + 6b^2 - 2c^2},$$

$$CC_2 = \frac{1}{3}\sqrt{6a^2 + 3b^2 - 2c^2}$$

- For the trisector points on the side BC, the lengths of AA_1 and AA_2 satisfy:

$$AA_1 = \frac{1}{3}\sqrt{3b^2 + 6c^2 - 2a^2}$$

$$AA_2 = \frac{1}{3}\sqrt{6b^2 + 3c^2 - 2a^2}$$

Method 3. Using Stewart's Theorem

With $AB_1 = B_1B_2 = B_2C = \frac{b}{3}$, then $AB_2 = B_1C = \frac{2b}{3}$.

By applying Stewart's Theorem to a triangle ABC with the cevian line BB_1, we obtain:

$$AB^2 \cdot B_1C + BC^2 \cdot AB_1 = AC\left(BB_1^2 + AB_1 \cdot B_1C\right) \tag{11}$$
$$c^2\left(\frac{2b}{3}\right) + a^2\left(\frac{b}{3}\right) = b\left(BB_1^2 + \left(\frac{b}{3}\right)\left(\frac{2b}{3}\right)\right)$$
$$\frac{2c^2b}{3} + \frac{a^2b}{3} = b\left(BB_1^2 + \left(\frac{2b^2}{9}\right)\right)$$
$$b\left(\frac{2c^2}{3} + \frac{a^2}{3}\right) = b\left(BB_1^2 + \left(\frac{2b^2}{9}\right)\right)$$
$$\frac{2c^2}{3} + \frac{a^2}{3} = BB_1^2 + \frac{2b^2}{9}$$
$$BB_1^2 = \frac{3a^2 - 2b^2 + 6c^2}{9}$$

Thus,

$$BB_1 = \frac{1}{3}\sqrt{3a^2 - 2b^2 + 6c^2}. \tag{12}$$

Similarly, using the same steps as in (12) for line BB_2, we also obtain:

$$BB_2 = \frac{1}{3}\sqrt{6a^2 - 2b^2 + 3c^2}. \tag{13}$$

By following the same steps as in (12) and (13), we find that:

- For the trisector points on the side AB, the lengths of CC_1 and CC_2 satisfy:

$$CC_1 = \frac{1}{3}\sqrt{3a^2 + 6b^2 - 2c^2}$$
$$CC_2 = \frac{1}{3}\sqrt{6a^2 + 3b^2 - 2c^2}$$

- For the trisector points on the side BC, the lengths of AA_1 and AA_2 satisfy:

$$AA_1 = \frac{1}{3}\sqrt{3b^2 + 6c^2 - 2a^2}$$
$$AA_2 = \frac{1}{3}\sqrt{6b^2 + 3c^2 - 2a^2}$$

Method 4. Using the Pythagorean Theorem

Consider triangle ABC in Figure 3. Draw a perpendicular line from vertex $\angle BAC = \beta$ that meets at point P.

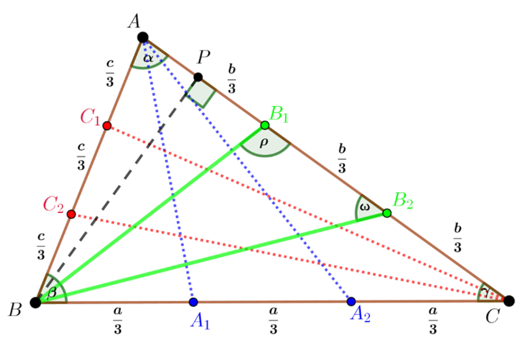

FIGURE 3. Side trisectors with the Pythagorean Theorem

In the right-angled triangle ABP, according to the Pythagorean theorem:

$$AB^2 = BP^2 + AP^2$$
$$c^2 = BP^2 + AP^2 \tag{14}$$

Thus,

$$BP^2 = c^2 - AP^2 \tag{15}$$

Furthermore, in the right-angled triangle BCP, we also obtain:

$$BC^2 = CP^2 + BP^2 \tag{16}$$

Since $CP = b - AP$, equation (16) becomes:

$$a^2 = (b - AP)^2 + BP^2 \tag{17}$$

Subtracting (14) from (17) to eliminate BP^2 yields:

$$a^2 - c^2 = (b - AP)^2 - AP^2$$
$$2bAP = b^2 + c^2 - a^2$$

So that,

$$AP = \frac{b^2 - a^2 + c^2}{2b} \tag{18}$$

Next, in the right-angled triangle BB_1P, it follows that:

$$BB_1^2 = BP^2 + B_1P^2 \tag{19}$$

Because B_1 lies on AC and $AB_1 = \frac{b}{3}$, then:

$$B_1P = AB_1 - AP$$
$$B_1P = \frac{b}{3} - AP \tag{20}$$

Substituting (15) and (20) into equation (19) gives:

$$BB_1^2 = BP^2 + B_1P^2$$
$$BB_1^2 = (c^2 - AP^2) + \left(\frac{b}{3} - AP\right)^2$$
$$BB_1^2 = c^2 - AP^2 + \frac{b^2}{9} - \frac{2b}{3}AP + AP^2$$
$$BB_1^2 = c^2 + \frac{b^2}{9} - \frac{2b}{3}AP \tag{21}$$

By substituting (18) into (21), we obtain:

$$BB_1^2 = c^2 + \frac{b^2}{9} - \frac{2b}{3}\left(\frac{b^2 - a^2 + c^2}{2b}\right)$$
$$BB_1^2 = c^2 + \frac{b^2}{9} - \frac{b^2}{3} + \frac{a^2}{3} - \frac{c^2}{3} = \frac{3a^2 - 2b^2 + 6c^2}{9}$$

Therefore,

$$BB_1 = \frac{1}{3}\sqrt{3a^2 - 2b^2 + 6c^2}. \tag{22}$$

Similarly, using the same steps as in (22) for triangle BB_2P, we find:

$$BB_2 = \frac{1}{3}\sqrt{6a^2 - 2b^2 + 3c^2}. \tag{23}$$

By following the same steps as in (22) and (23), we find that:

- For the trisector points on the side AB, the lengths of CC_1 and CC_2 satisfy:

$$CC_1 = \frac{1}{3}\sqrt{3a^2 + 6b^2 - 2c^2}$$

$$CC_2 = \frac{1}{3}\sqrt{6a^2 + 3b^2 - 2c^2}$$

- For the trisector points on the side BC, the lengths of AA_1 and AA_2 satisfy:

$$AA_1 = \frac{1}{3}\sqrt{3b^2 + 6c^2 - 2a^2}$$

$$AA_2 = \frac{1}{3}\sqrt{6b^2 + 3c^2 - 2a^2}$$

Therefore, Theorem 3.1 is proven. □

Theorem 3.2. *In any triangle ABC with $BC = a$, $AC = b$, $AB = c$, let the side trisectors be $BB_1 = p$ and $BB_2 = q$, which divide side b such that $AB_2 = B_2B_1 = B_1C = r = \frac{b}{3}$. If a tangential excircle is constructed on one of the sides of the triangle resulting from the side trisection, namely the triangle BB_1B_2, then the radius of the tangential excircle on side B_2B_1 is given by:*

$$OW = OG = OF = R_{TB} = \frac{L\triangle ABC}{3(s_{\triangle BB_1B_2} - r)}$$

Proof. Theorem 3.2 can be illustrated in Figure 4. The trisector points B_1 and B_2 divide the side AC into three equal parts. Therefore, the area of $\triangle BB_1B_2$ is one-third of the area of $\triangle ABC$, namely:

$$L\triangle BB_1B_2 = \frac{L\triangle ABC}{3} \tag{24}$$

For $\triangle BB_1B_2$ with sides $BB_1 = p$, $BB_2 = q$, $B_1B_2 = r$, the following holds:

$$s_{\triangle BB_1B_2} = \frac{1}{2}(p + q + r) \tag{25}$$

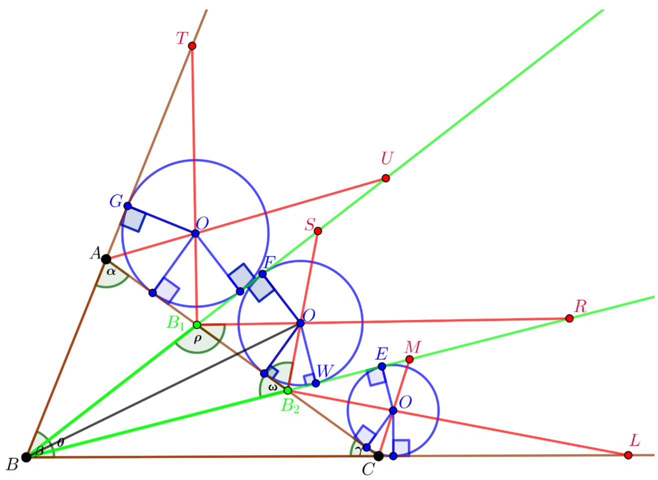

Figure 4. Tangential excircle of the triangle's side trisector

Consider the quadrilateral BB_1OB_2 in Figure 4. The area of this region can be expressed as the sum of the areas of two right-angled triangles:

$$L\square BB_1OB_2 = L\triangle BB_1O + L\triangle BB_2O = \frac{1}{2} \cdot BB_1 \cdot OF + \frac{1}{2} \cdot BB_2 \cdot OW \tag{26}$$

$$L\square BB_1OB_2 = \frac{1}{2} \cdot p \cdot R_{TB} + \frac{1}{2} \cdot q \cdot R_{TB} \tag{27}$$

On the other hand, the same area can also be written as:

$$L\square BB_1OB_2 = L\triangle BB_1B_2 + L\triangle B_1B_2O$$

Substituting (24), yields:

$$L_{BB_1OB_2} = \frac{L_{\triangle ABC}}{3} + \frac{1}{2}\cdot r\cdot R_{T_B} \tag{28}$$

By substituting equation (27) into equation (28), we obtain:

$$\frac{1}{2}\cdot p\cdot R_{TB} + \frac{1}{2}\cdot q\cdot R_{TB} = \frac{L\triangle ABC}{3} + \frac{1}{2}\cdot r\cdot R_{TB}$$

Grouping the terms containing R_{TB} results in:

$$\begin{aligned}\frac{1}{2}p\cdot R_{TB} + \frac{1}{2}q\cdot R_{TB} - \frac{1}{2}r\cdot R_{TB} &= \frac{L\triangle ABC}{3}\\ R_{TB}\left(\frac{1}{2}\cdot p + \frac{1}{2}\cdot q + \frac{1}{2}\cdot r\right) &= \frac{L\triangle ABC}{3}\\ R_{TB}\left(\frac{1}{2}(p+q-r)\right) &= \frac{L\triangle ABC}{3}\\ R_{TB}\left(\frac{1}{2}(p+q+r)-r\right) &= \frac{L\triangle ABC}{3} \qquad (29)\end{aligned}$$

Substituting (25) into equation (29) leads to:

$$R_{TB}\left(s_{\triangle BB_1B_2} - r\right) = \frac{L\triangle ABC}{3}$$

Thus,

$$R_{TB} = \frac{L\triangle ABC}{3(s_{\triangle BB_1B_2} - r)}$$

Using the same method for $\triangle ABB_1$ and $\triangle BCB_2$, we also obtain:

- The radius of the tangential excircle on the side $AB_1 = R_{TA} = \dfrac{L\triangle ABC}{3(s_{\triangle ABB_1} - r)}$
- The radius of the tangential excircle on the side $B_2C = R_{TC} = \dfrac{L\triangle ABC}{3(s_{\triangle BCB_2} - r)}$

Therefore, Theorem 3.2 is proven. □

4. Conclusion

The side lengths created by trisecting a triangle's sides have been derived using several alternative proofs. This study shows that these side trisector lengths have a consistent mathematical relationship with the triangle's original sides. These formulas also allow calculation of the radius of the tangential excircles in the triangle formed by side trisection.

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